AP® Calculus AB
2002 Sample Student Responses
Form B

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1 1 1 1 1 1 1 1 1 1 A,

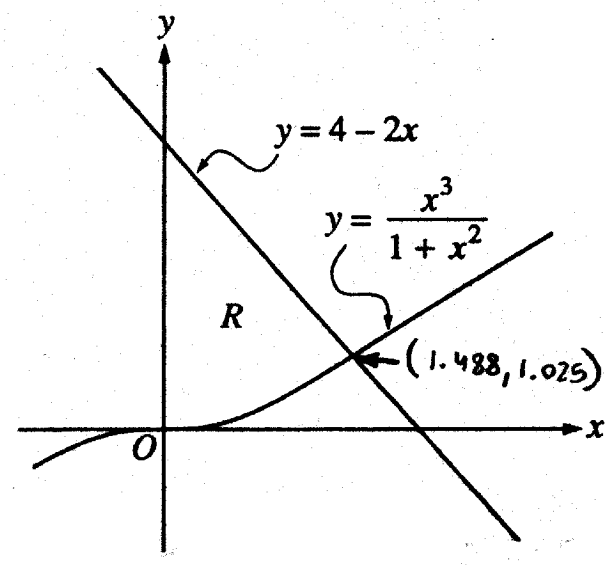
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



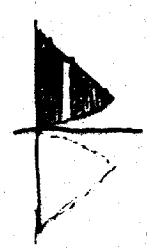
Work for problem 1(a)

Limits of Integration: $4-2x = \frac{x^3}{1+x^2}$

$x = 0, 1.488$

$R = \int_0^{1.488} \left[4-2x-\left(\frac{x^3}{1+x^2}\right)\right] dx \approx \boxed{3.215}$

Work for problem 1(b)



$r_1 = 4 - 2x$

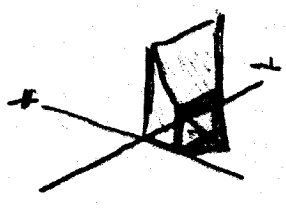
$r_2 = \frac{x^3}{1+x^2}$

$V_R = \int \pi(r_1^2 - r_2^2)\,dx$

$= \pi \int_0^{1.488} \left[(4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right] dx$

$V_R \approx \boxed{31.885}$

Work for problem 1(c)



$A_\square = s^2$

$s = (4-2x) - \left(\frac{x^3}{1+x^2}\right)$

$V_R = \int s^2\,dx = \int_0^{1.488} \left[4-2x-\left(\frac{x^3}{1+x^2}\right)\right]^2 dx$

$\approx \boxed{8.997}$

GO ON TO THE NEXT PAGE.

1 1 1 1 1 1 1 1 1 1 C,

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

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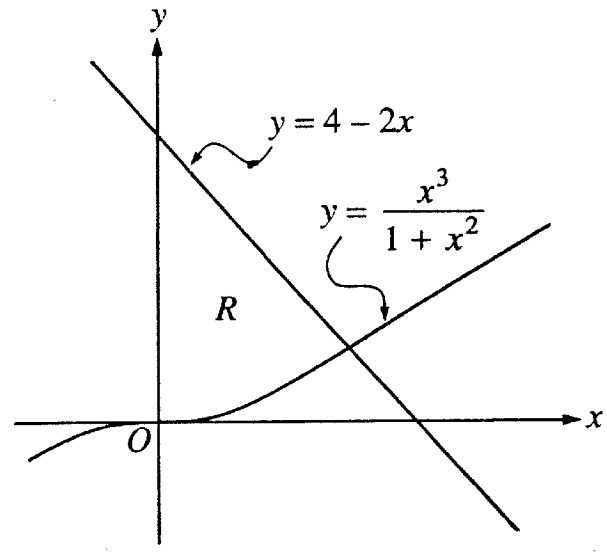


Work for problem 1(a)

y_1 intersects y_2 at 1.4877

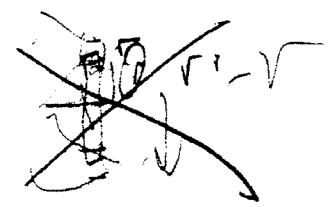
$$\int_0^{1.4877} 4-2x \;-\; \int_0^{1.4877} \frac{x^3}{1+x^2}$$

$= 3.7375 - 0.5229$ (used calculator)

$= 3.215$ units^2

Work for problem 1(b)

$\Delta V = \pi(r_1^2 - r_2^2)\Delta x$



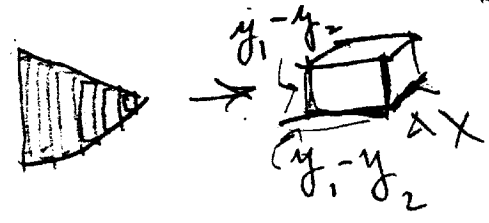
$\Delta V = \pi\left[(4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right]\Delta x$

$V = \pi\int_0^{1.4997}\left[(4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right]dx$

$V = 320.694$ units^3 (used calculator)

Work for problem 1(c)

~~Cross section~~ Cross section



$\Delta V = (y_1 - y_2)^2\Delta x$

$\Delta V = \left[(4-2x) - \left(\frac{x^3}{1+x^2}\right)\right]^2\Delta x$

$V = \int_0^{1.4997}\left[(4-2x) - \left(\frac{x^3}{1+x^2}\right)\right]^2 dx$

$V = 100.803$ units^3 (used calculator)

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A1

Work for problem 2(a)

a) $P'(9) = 1 - 3(e)^{-0.2\sqrt{9}}$

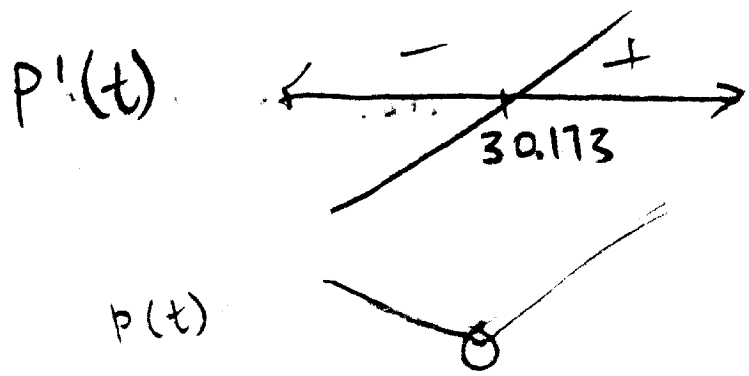
$= -0.646$ gallons/day.

No. $P'(9)$ is negative, so the amount of pollutant is decreasing.

Work for problem 2(b)

b) $p'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$

$t = 30.173$



minimum at $t = 30.173$

A_2

Work for problem 2(c)

$$50 + \int_0^{30.173} P'(t)\,dt$$

$$= 50 - 14.895$$

$$= 35.104 \text{ gallons}$$

At day 30, there will be 35.104 gallons of pollutant left, and $35.104 < 40$, ∴ it will be safe.

Work for problem 2(d)

$$P'(0) = 1 - 3e^{-0.2(0)}$$

$$= 1 - 3e^{0}$$

$$= 1 - 3$$

$$= -2$$

$P(0) = 50$

$y - 50 = -2(x)$

$y = -2x + 50$

$y \leq 40$

$-2x + 50 \leq 40$

$-2x \leq -10$

$x \geq 5$

It predicts that at $t = 5$ the lake will become safe.

GO ON TO THE NEXT PAGE.

C1

Work for problem 2(a)

$P'(9) = 1 - 3e^{-0.2\sqrt{9}} = -0.646$ gallons/day

the level of pollutant is decreasing because the rate is negative, as it is decreasing.

Work for problem 2(b)

gallons of pollutant at a min when $P'(t) = 0$

$1 - 3e^{-0.2\sqrt{t}} = 0$

$3e^{-0.2\sqrt{t}} = 1$

$e^{-0.2\sqrt{t}} = 1/3$

$-0.2\sqrt{t} = \ln 1/3$

$\sqrt{t} = \frac{\ln 1/3}{-0.2}$

$t = \left(\frac{\ln 1/3}{-0.2}\right)^2$

$= 30.174$

≈ 30 days

C_2

Work for problem 2(c)

no. of gallons present at the lake

$$= 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}})\,dt$$

$$= 50.000 \text{ gallons.}$$

the lake is not safe because the no. of gallons is above 40 gallons.

Work for problem 2(d)

slope of tangent $= 1 - 3e^{-0.2\sqrt{t}}$

at $t=0$; gallons $= 50 \Rightarrow m_T\big|_{(0,50)} = 1 - 3e^{-0.2\sqrt{0}} = -2$

equation of tangent: $y = -2x + 50$

40 lake is safe $\Rightarrow 40 = -2x + 50$

$\Rightarrow -2x = -10$

$\Rightarrow x = 5$

after 5 days.

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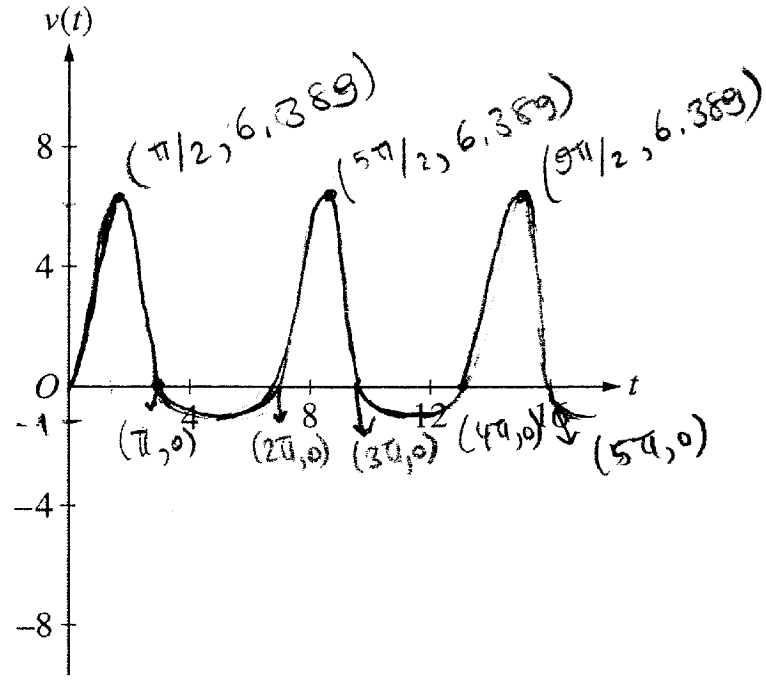
B1

Work for problem 3(a)

$v(t) = e^{2\sin(t)} - 1$

$x(0) = 0$



Work for problem 3(b)

If we consider y^+ to be the $\xrightarrow{x^+}$ (right)

then between

$\pi < x < 2\pi$, $3\pi < x < 4\pi$ and $5\pi < x < 16$

the particle is going to left.

(when $v(t)$ is $\underline{\underline{(-)}}$)

B_2

Work for problem 3(c)

$$\text{distance}_{\text{total}} = \int_0^4 |v(t)|\,dt = \int_0^4 |e^{2\sin t} - 1|\,dt$$

$$= 10.542$$

Work for problem 3(d)

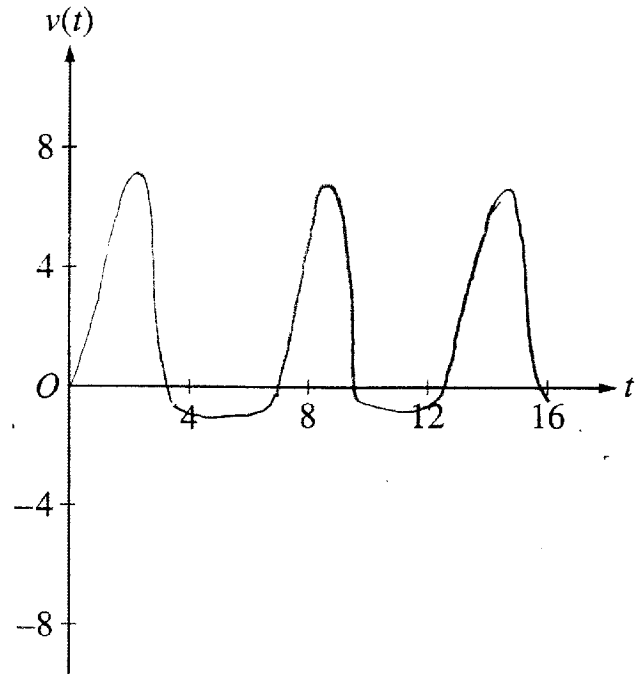
NO; since the area below the x-axis is not equal to the area above the x-axis, the particle will not return to the origin at any time:

$$\int_{\pi}^{2\pi} v(t)\,dt + \int_{3\pi}^{4\pi} v(t)\,dt + \int_{5\pi}^{6} v(t)\,dt < \int_{0}^{\pi} v(t)\,dt + \int_{2\pi}^{3\pi} v(t)\,dt + \int_{4\pi}^{5\pi} v(t)\,dt$$

3 3 3 3 3 3 3 3 3 3

C1

Work for problem 3(a)



Work for problem 3(b)

When $v(t)$ is less than zero, the particle moves to the left.

$e^{2\sin t} - 1 = 0$

$e^{2\sin t} = 1$

$2\sin t = \ln 1$

$2\sin t = 0$

$\sin t = 0$

$t = 0, \pi + n\pi$

Velocity is < 0 on $(\pi, 2\pi)(3\pi, 4\pi)(5\pi, 16)$, therefore the particle is moving left on these intervals

C2

Work for problem 3(c)

Total distance travelled = $\int_0^4 |v(t)|\,dt$

$D = \int_0^4 |e^{2\sin t} - 1|\,dt$

by graphing calculator:

$D = 8.23$

Work for problem 3(d)

When the particle is at the origin

$0 = \int_0^t v(t)\,dt$

by graphing calculator:

the area of intervals above x-axis is always greater than those below so the particle never returns to the origin. The movement is not sustained for long enough to bring it back to the origin.

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4 4 4 4 4 4 4 4 4 4

A,

NO CALCULATOR ALLOWED

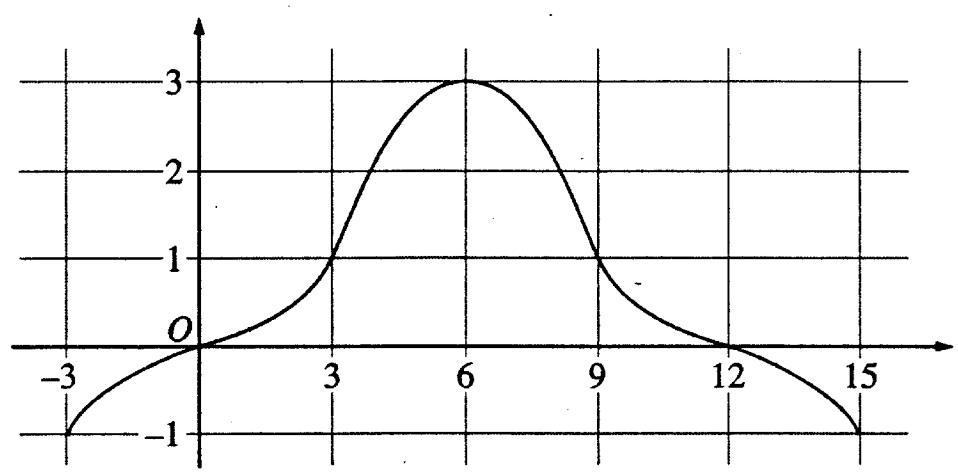
CALCULUS

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(6) = 5 + \int_6^6 f(t)\,dt = 5$

$g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

Work for problem 4(b)

$g'(x) = \frac{d}{dx}\int_6^x f(t)\,dt = f(x)$

g decreases when $f(x) < 0$.

$-3 < x < 0$, $12 < x < 15$

NO CALCULATOR ALLOWED

A_2

Work for problem 4(c)

$g''(x) = f'(x) < 0$

$f'(x) < 0$ when $f(x)$ is decreasing

$6 < x < 15$

Work for problem 4(d)

$3 \times \left(\frac{-1+0}{2}\right) + 3 \times \left(\frac{0+1}{2}\right) + 3 \times \left(\frac{1+3}{2}\right) + 3 \times \left(\frac{3+1}{2}\right) + 3 \times \left(\frac{1+0}{2}\right) + 3 \times \left(\frac{0+(-1)}{2}\right)$

$= 3 \times 4 = 12$

GO ON TO THE NEXT PAGE.

4 4 4 4 4 4 4 4 4 4 C,

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

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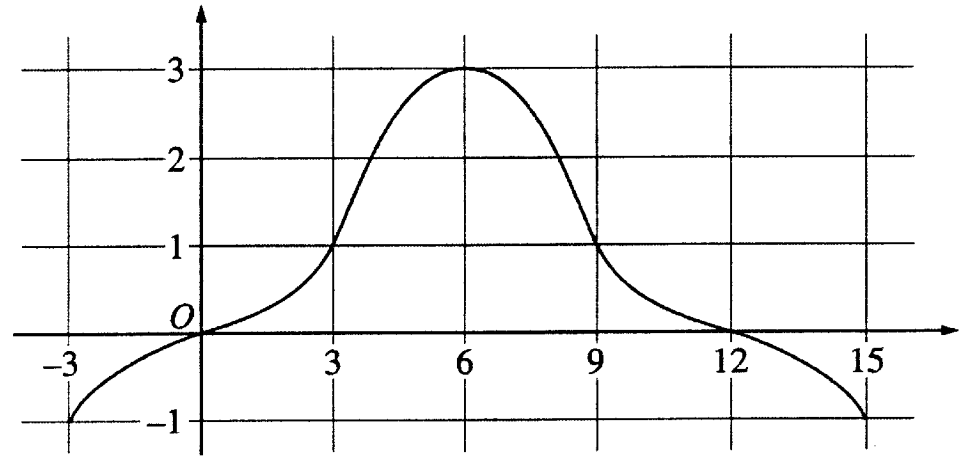

Graph of f

Work for problem 4(a)

$g(6) = 5 + \int_6^6 f(t)\,dt = 0$

$g'(x) = f(x)$

$\therefore g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

Work for problem 4(b)

$g'(x) = f(x)$ from $g'(x) = 0 + \frac{d}{dx}\left[\int_6^x f(t)\,dt\right]$

$f(x) < 0$ on $-3 < t < 0$ and $12 < t < 15$

$\therefore g(x)$ is decreasing on $-3 < t < 0$ and $12 < t < 15$

NO CALCULATOR ALLOWED

C_2

Work for problem 4(c)

$g''(x) = f'(x)$

$f'(x) < 0$ on $6 < t < 15$

$\therefore g(x)$ is concave down on $6 < t < 15$

Work for problem 4(d)

$A \approx \frac{18}{12}(|-1| + (1)(2) + (3)(2) + (1)(2) + |-1|)$

$\approx \frac{18}{12}(12) \approx 18$ squared units

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A1

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

Work for problem 5(a)

1) Solving the equation

$\frac{dy}{dx} = \frac{3-x}{y}$

$y\,dy = (3-x)\,dx$

$\int y\,dy = \int (3-x)\,dx$

$\frac{y^2}{2} = 3x - \frac{x^2}{2} + C \qquad y^2 = 6x - x^2 + C$. General solution.

2) Because $y=-2$ is tangent to $f(x)$ at $(x_0, -2)$,

$\left.\frac{dy}{dx}\right|_{\substack{x=x_0 \\ y=-2}} = 0. \qquad \frac{3-x_0}{-2} = 0$

$x_0 = 3.$

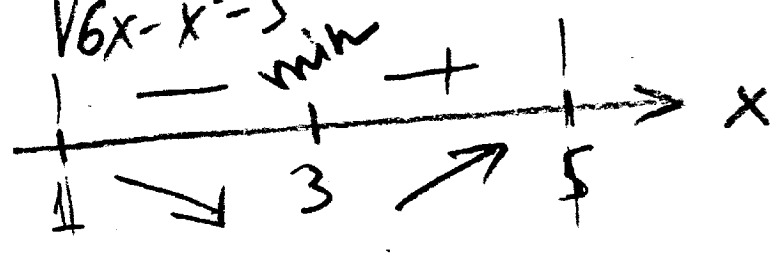
3) Particular solution $y = -\sqrt{6x \cdot}$

$(3,-2) \quad 4 = 6\cdot 3 - 9 + C$

$C = \bullet - 5.$

4) $y' = \frac{-(3-x)}{\sqrt{6x-x^2-5}} = \frac{x-3}{\sqrt{6x-x^2-5}}$

at $x=3$



at $x=3$ $y=f(x)$ has a local minimum.

A2

NO CALCULATOR ALLOWED

Work for problem 5(b)

$$\frac{dy}{dx} = \frac{3-x}{y}$$

$y^2 = 6x - x^2 + C$ as found in a)1).

$y(6) = -4$ $\quad 16 = 6\cdot(+6) - 36 + C$

$\therefore C = 16$

$y^2 = 6x - x^2 + 16$

$y = -\sqrt{6x - x^2 + 16}$

GO ON TO THE NEXT PAGE.

D1

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

Work for problem 5(a)

14 x 15

$y' = \frac{3-x}{y}$

$\frac{dy}{dx} = \frac{3-x}{y}$

$0 = \frac{3-x}{y}$

$0 = 3 - x$

~~Point of tangency~~ $x = 3$

$3 = x$ - coordinate of point of tangency, local maximum

$y = \pm\sqrt{6x - x^2}$

$x = 0$
$y = 0$

$x = 1 \quad y = \sqrt{5}$

$x = 4 \quad y = 8$

inc

$\frac{2}{\sqrt{5}}$

I	$f'(x)$
$-\infty, 3$	inc
$(3, \infty)$	dec

$\frac{-1}{8}$ c

$y = \pm\sqrt{6 - 1}$

D2

NO CALCULATOR ALLOWED

Work for problem 5(b)

(6, -4)

$\int y\,dy = \int 3-x\,dx$

$\frac{1}{2}y^2 =$

$\int y\,dy = \int 3-x\,dx$

$\frac{1}{2}y^2 + C = 3x - \frac{1}{2}x^2 + C$

$y = \sqrt{6x - x^2 + C}$

$-4 = \sqrt{36 - 36 + C}$

$-4 = \pm\sqrt{C}$

$g(x) = -\sqrt{6x - x^2 + 16}$

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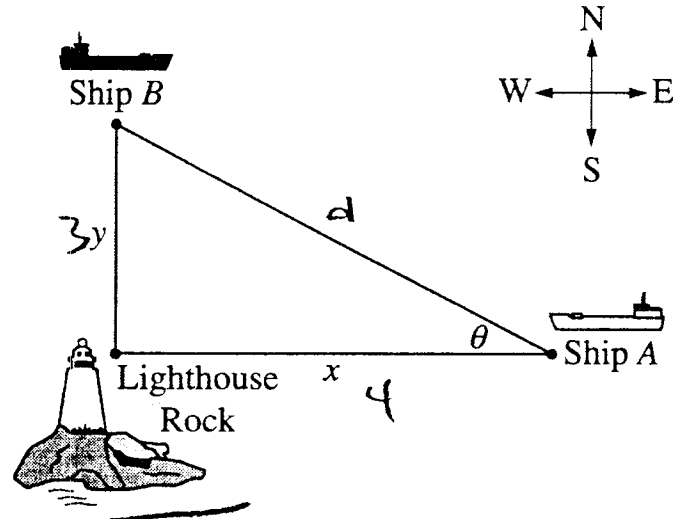
B1



Work for problem 6(a)

$d = \sqrt{x^2 + y^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$ km

Work for problem 6(b)

$d^2 = x^2 + y^2$

$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$(10)\frac{dd}{dt} = (8)(-15) + (6)(10)$

$= \frac{-120 + 60}{10}$

$= -6$ km/h

NO CALCULATOR ALLOWED

B2

Work for problem 6(c)

$y = d \sin\theta$

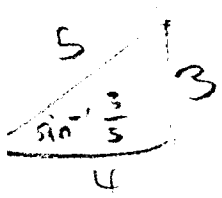
$\frac{dy}{dt} = d\cos\theta \frac{d\theta}{dt} + \frac{dd}{dt}\sin\theta$

$\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$10 = (5)\cos\theta \frac{d\theta}{dt} + (-6)\sin\theta$

$= 5\cos\left[\sin^{-1}\frac{3}{5}\right]\frac{d\theta}{dt} + (-6)\sin\left[\sin^{-1}\frac{3}{5}\right]$



$= 5\left(\frac{4}{5}\right)\frac{d\theta}{dt} - 6\left(\frac{3}{5}\right)$

$= 4\frac{d\theta}{dt} - \frac{18}{5}$

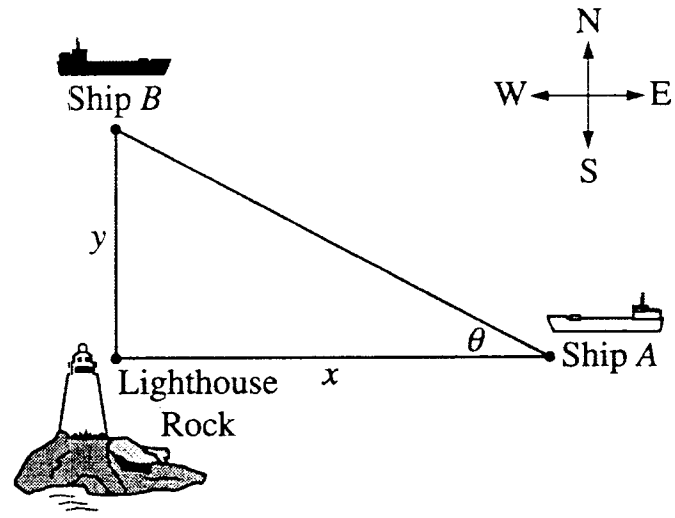
$\frac{d\theta}{dt} = \frac{50 + 18}{5} \cdot \frac{1}{4}$

$\frac{d\theta}{dt} = \frac{68}{20}$

$= \frac{17}{5}$ rad/hour

NO CALCULATOR ALLOWED

C1



Work for problem 6(a)

$x^2 + y^2 = z^2$

$(4)^2 + (3)^2 = z^2$

$16 + 9 = z^2$

$25 = z^2$

$5 = z$

5 km away

Work for problem 6(b)

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(4)(15) + 2(3)(10) = 2(5)\left(\frac{dz}{dt}\right)$

$120 + 60 = 10\frac{dz}{dt}$

$\frac{180}{10} = \frac{dz}{dt}$

18 km/hr

NO CALCULATOR ALLOWED

C2

Work for problem 6(c)

$\tan\theta = \frac{y}{x}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{x\left(\frac{dy}{dt}\right) - (y)\left(\frac{dx}{dt}\right)}{x^2}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{4(15) - (3)(10)}{4^2}$

$= \frac{60 - 30}{16}$

$= \frac{30}{16}$

$\frac{d\theta}{dt} = \frac{30}{16\sec^2\theta}$ radians/hour

$\tan^{-1}\theta = \frac{3}{4}$

$\tan\frac{3}{4} = \theta$

$\frac{d\theta}{dt} = \frac{30}{16\sec^2}$